Hutton Church of England Grammar School and Sixth Form

Maths Curriculum Information, Intent and Map

Curriculum Vision

Mathematics is part of everyday life and work and is therefore an essential skill. It provides a means of communication which is powerful, concise and unambiguous. It helps develop powers of logical thinking, accuracy and spatial awareness.

Mathematics is not only taught because it is useful; it should be seen as a delight and wonder, offering pupils intellectual excitement and an appreciation of its essential creativity.

Mathematics is an essential qualification and is a passport to employment and further education. We have challenging targets and high expectations for all our pupils to help them succeed.

We aim to smooth the transition for all pupils between the Key Stages and ensure progression in teaching and learning throughout their time at School.

In the delivery of Mathematics at Key Stage 3, the GCSE and GCE syllabi at Key Stages 4 and 5, the Department aims to develop in our pupils and students:

1. A positive attitude towards mathematics and an awareness of its fascination.
2. An understanding of mathematics through a process of enquiry, experiment and listening to others.
3. Competence and confidence in mathematical knowledge, concepts and skills.
4. An ability to solve problems, to reason, to think logically and to work systematically and logically.
5. Initiative and an ability to work both independently and in co-operation with others.
6. An ability to communicate through mathematics.
7. An ability to use and apply mathematics across the curriculum and in real life.

'And let us not grow weary of doing good, for in due season we will reap, if we do not give up'
Galatians 6:9

Mathematics programmes of study: Key Stage 3 National Curriculum in England

Purpose of study

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Aims: Key Stage 3

The national curriculum for mathematics aims to ensure that all pupils:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

- Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
- Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programme of study for key stage 3 is organised into apparently distinct domains, but pupils should build on key stage 2 and connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge in science, geography, computing and other subjects.

Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content in preparation for key stage 4. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on. Information and communication technology (ICT) Calculators should not be used as a substitute for good written and mental arithmetic. In secondary schools, teachers should use their judgement about when ICT tools should be used.

Spoken language

The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.

Attainment targets

By the end of key stage 3, pupils are expected to know, apply and understand the matters, skills and processes specified in the relevant programme of study.

Working mathematically

Through the mathematics content, pupils should be taught to:

1. <u>Develop fluency</u>

- Consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- Select and use appropriate calculation strategies to solve increasingly complex problems
- Use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- Substitute values in expressions, rearrange and simplify expressions, and solve equations
- Move freely between different numerical, algebraic, graphical and diagrammatic representations

- Develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- Use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics.

2. Reason mathematically

- Extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- Extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- Identify variables and express relations between variables algebraically and graphically
- Make and test conjectures about patterns and relationships; look for proofs or counterexamples
- Begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- Interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- Explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally.

3. Solve problems

- Develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- Develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- Begin to model situations mathematically and express the results using a range of formal mathematical representations
- Select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

4. Number

- Understand and use place value for decimals, measures and integers of any size
- Order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, , ≤, ≥
- Use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- Use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- Use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- Recognise and use relationships between operations including inverse operations
- Use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- Interpret and compare numbers in standard form A x 10n 1≤A

Mathematics programmes of study: Key Stage 4 National Curriculum in England

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programme of study for key stage 4 is organised into apparently distinct domains, but pupils should develop and consolidate connections across mathematical ideas. They should build on learning from key stage 3 to further develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge wherever relevant in other subjects and in financial contexts.

The expectation is that the majority of pupils will move through the programme of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

This programme of study specifies:

- The mathematical content that should be taught to all pupils, in standard type; and
- **Additional mathematical content to be taught to more highly attaining pupils, in bold type and braces { }. These pupils are likely to be entered for the Higer GCSE Paper.**

Together, the mathematical content set out in the key stage 3 and key stage 4 programmes of study covers the full range of material contained in the GCSE Mathematics qualification. Wherever it is appropriate, given pupils' security of understanding and readiness to progress, pupils should be taught the full content set out in this programme of study

Working Mathematically

Through the mathematics content, pupils should be taught to:

1. Develop fluency

- Consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include powers, roots **{and fractional indices}**
- Select and use appropriate calculation strategies to solve increasingly complex problems, including exact calculations involving multiples of π **{and surds},** use of standard form and application and interpretation of limits of accuracy
- Consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions, **{and expressions involving surds and algebraic fractions}**
- Extend fluency with expressions and equations from key stage 3, to include quadratic equations, simultaneous equations and inequalities
- Move freely between different numerical, algebraic, graphical and diagrammatic representations, including of linear, quadratic, reciprocal, **{exponential and trigonometric}** functions
- Use mathematical language and properties precisely.

2. Reason mathematically

- Extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically.
- Extend their ability to identify variables and express relations between variables algebraically and graphically.
- Make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments **{and proofs}.**
- Reason deductively in geometry, number and algebra, including using geometrical constructions.
- Interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning.
- Explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally.
- Assess the validity of an argument and the accuracy of a given way of presenting information.

3. Solve problems

- Develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems.
- Develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts.
- Make and use connections between different parts of mathematics to solve problems.
- Model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions.
- Select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem.

Subject Content

Number

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- apply systematic listing strategies, **{including use of the product rule for counting}**
- **{estimate powers and roots of any given positive number}**
- calculate with roots, and with integer **{and fractional}** indices
- calculate exactly with fractions, **{surds}** and multiples of π; **{simplify surd expressions involving squares and rationalise denominators}**
- calculate with numbers in standard form A 10n , where 1 ≤ A < 10 and n is an integer
- **{change recurring decimals into their corresponding fractions and vice versa}**
- identify and work with fractions in ratio problems
- apply and interpret limits of accuracy when rounding or truncating, **{including upper and lower bounds}.**

Algebra

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- simplify and manipulate algebraic expressions (including those involving **surds {and algebraic fractions}**) by:
    - factorising quadratic expressions of the form 2 x bx c + + 2 ax bx c + + , including the difference of two squares; **{factorising quadratic expressions of the form}**
    - simplifying expressions involving sums, products and powers, including the laws of indices
- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- where appropriate, interpret simple expressions as functions with inputs and outputs; **{interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}**
- use the form y mx c = + to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically **{and turning points by completing the square}**
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function 1 y = x y x = cos with x ≠ 0, **{the exponential function x y k = y x = sin for positive values of k, and the trigonometric functions (with arguments in degrees) , and y x = tan for angles of any size}**
- **{sketch translations and reflections of the graph of a given function}**
- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**
- **{recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point}**
- solve quadratic equations **{including those that require rearrangement}** algebraically by factorising, **{by completing the square and by using the quadratic formula}**; find approximate solutions using a graph
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- **{find approximate solutions to equations numerically using iteration}**
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- solve linear inequalities in one **{or two}** variable{s}, **{and quadratic inequalities in one variable}; represent the solution set on a number line, {using set notation and on a graph}**

- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r n where n is an integer, and r is a positive rational number **{or a surd}**) **{and other sequences}**
- deduce expressions to calculate the nth term of linear **{and quadratic}** sequences

Ratio, proportion and rates of change

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)
- convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- understand that X is inversely proportional to Y is equivalent to X is proportional to 1 Y; **{construct and}** interpret equations that describe direct and inverse proportion
- interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- **{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}**
- set up, solve and interpret the answers in growth and decay problems, including compound interest **{and work with general iterative processes}.**

Geometry and measures

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- interpret and use fractional **{and negative}** scale factors for enlargements
- **{describe the changes and invariance achieved by combinations of rotations, reflections and translations}**
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- **{apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results}**
- construct and interpret plans and elevations of 3D shapes
- interpret and use bearings
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles **{and, where possible, general triangles}** in two **{and three}** dimensional figures
- know the exact values of sin cos θ θ and 0 0 0 θ = 0, 30, 45 60 0 and sin sin sin abc ABC = = for 0 0 0 0 θ = 0, 30, 45, 60 90 0 and; know the exact value of tanθ a2 = b2 2 + − c 2bc cos A 1 Area = sin 2 ab C for
- **{know and apply the sine rule, , and cosine rule, , to find unknown lengths and angles}**
- **{know and apply to calculate the area, sides or angles of any triangle}**
- describe translations as 2D vectors

- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}.**

Probability

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**

Statistics

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- interpret and construct tables and line graphs for time series data
- **{construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}**
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
    - appropriate graphical representation involving discrete, continuous and grouped data**, {including box plots}**
    - appropriate measures of central tendency (including modal class) and spread **{including quartiles and inter-quartile range}**
- apply statistics to describe a population
- use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

Progression

Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content in preparation for key stage 4. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on.

Curriculum Map

Key Stage 3

	Year 7	**Year 8**	**Year 9**
Half Term 1	Numbers and the number system Calculations	Numbers and number system Calculations BIDMAS, Standard form	Calculations Indices Roots Construction
Half Term 2	Approximation Estimating Comparing Fractions, Decimals, %'s	Angles, Bearings Probability Algebra	Algebra Proportion
Half Term 3	Shape Algebra Formulae	Fractions Decimals, %'s Ratio Proportional reasoning Number Patterns	Number Patterns Equations Inequalities
Half Term 4	Ratio Proportional reasoning Measuring space number patterns	Angle properties Algebra Measuring space	Calculating space Congruent triangles Similar triangles Pythagoras
Half Term 5	Angles Fractions, Decimals, %'s Equations	Graphs Probability	Algebra-Graphs
Half Term 6	Measuring and presentation of data	Measuring and presentation of data	Probability Statistics

Curriculum Map

Key Stage 4

We follow the GCSE AQA Specification B

		Year 10	Year 11
Half Term 1	Foundation	Angles, Bearings, Basic number, Factors, Multiples, Basic Algebra, Basic Fractions, Coordinates and graphs	Probability, Volume, Algebra, Scatter Graphs
	Higher	Angles, Bearings, Number, Algebra, Fractions & decimals, Coordinates & graphs	Probability, Volume, Algebra, Scatter graphs, Numerical methods
Half Term 2	Foundation	Basic decimals, Rounding, Collecting and representing data sequences	Inequalities, Pythagoras, Simultaneous equations, Algebra & graphs
	Higher	Rounding, Sequences, Percentages,	Equation of a circle, Equations and graphs, Simultaneous Equations
Half Term 3	Foundation	Basic percentages, Perimeter and area, Circles, Real life graphs	Algebra & graphs, Direct & Inverse Proportion
	Higher	Perimeter & area, Circles, Real life graphs, Ratio & proportion, Polygons	Sketching Graphs, Direct & Inverse Proportion, Inequalities, Pythagoras & Trigonometry
Half Term 4	Foundation	Ratio & Proportion, Polygons, Equations, Indices, Standard Form, Basic Probability	Trigonometry, Quadratic Equations, Quadratic Graphs
	Higher	Equations, Indices, Surds, Probability, Standard Form	Growth & Decay, Vectors, Transforming Graphs, Sine & Cosine Rule, Circle Theorems
Half Term 5	Foundation	Transformations, Congruency and similarity, 3D shapes, Percentages	Growth & Decay, Vectors Revision
	Higher	Transformations, Congruency & similarity, 3D shapes, Percentages	Rates of change, Area under a curve, Algebraic Fractions, Revision
Half Term 6	Foundation	Measures, Construction & Loci, Statistical Measures	GCSE Examinations
	Higher	Measures, Construction & Loci, Statistical Measures	GCSE Examinations

Curriculum Map

Key Stage 5

We follow the AQA A Level Specification for Mathematics

	Year 12	**Year 13**
Half Term 1	Algebraic Manipulation, Quadratic & Simultaneous Equations, Graphs, linear & Quadratic Inequalities, Straight Line & Circles,	Trigonometry, Functions, Coordinate Geometry
Half Term 2	Trigonometry, Exponential and Logarithms	Differentiation, Sequences and series
Half Term 3	Differentiation, Integration, Vectors, Binomial Expansion	Integration, Numerical methods, Mechanics
Half Term 4	Mechanics Statistics	Differential Equations Vectors, Proof, LDS
Half Term 5	Statistics, Mechanics,	Revision
Half Term 6	Proof, LDS	A Level Examinations